Examiners' Report: Final Honour School of Mathematics Part B Trinity Term 2021

January 28, 2022

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

  See Table 1.

	Numbers					Percentages %				
	2021	(2020)	(2019)	(2018)	(2017)	2021	(2020)	(2019)	(2018)	(2017)
I	51	(73)	(59)	(58)	(51)	39.84	(46.5)	(39.07)	(38.16)	(38.63)
II.1	58	(66)	(67)	(67)	(64)	45.31	(42.04)	(44.37)	(44.08)	(48.48)
II.2	18	(13)	(20)	(25)	(11)	14.06	(8.28)	(13.25)	(16.45)	(8.33)
III	1	(4)	(4)	(2)	(3)	0.78	(2.55)	(2.65)	(1.32)	(2.27)
P	1	(0)	(0)	(2)	(0)	0.64	(0)	(0)	(1.52)	(0)
F	0	(1)	(0)	(0)	(0)	0	(0.66)	(0)	(0)	(0)
Total	157	(151)	(152)	(132)	(141)	100	(100)	(100)	(100)	(100)

Table 1: Numbers and percentages in each class

- **Numbers of vivas and effects of vivas on classes of result.**

  As in previous years there were no vivas conducted for the FHS of Mathematics Part B.

- **Marking of scripts.**

  BE Extended Essays, BSP projects, and coursework submitted for the History of Mathematics course were double marked.

  The remaining scripts were all single marked according to a pre-agreed marking scheme which was strictly adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**

  See Table 5 on page 12.

B. New examining methods and procedure in the 2021 examinations

In light of the ongoing Covid 19 pandemic, the University changed the examinations to an open-book format and rolled out a new online examinations platform. An additional 30 minutes was added on to the exam duration to allow candidate the technical time to download and submit their examination papers via Inspera.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

The department agreed that the examinations would be in person in Trinity Term 2022.

D. Notice of examination conventions for candidates

The first Notice to Candidates was issued on 19 March 2021 and the second notice on 30 April 2021.

All notices and the examination conventions for 2021 are on-line at http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments.

Part II

A. General Comments on the Examination

The examiners would like to convey their grateful thanks for their help and cooperation to all those who assisted with this year's examination, either as assessors or in an administrative capacity. The chairman would particularly like to thank Elle Styler for administering the whole process with efficiency, and also to thank Nicole Collins, Charlotte Turner-Smith and Waldemar Schlackow.

In addition the internal examiners would like to express their gratitude to Professor Marco Schlichting and Professor Anne Skeldon for carrying out their duties as external examiners in a constructive and supportive way during the year, and for their valuable input at the final examiners' meetings.

Standard of performance

The standard of performance was broadly in line with recent years. In setting the USMs, we took note of

- the Examiners' Report on the 2020 Part B examination, and in particular recommendations made by last year's examiners, and the Examiners' Report on the 2020 Part A examination, in which the 2021 Part B cohort were awarded their USMs for Part A;
- a document issued by the Mathematics Teaching Committee giving broad guidelines on the proportion of candidates that might be expected in each class, based on the class percentages over the last five years in Mathematics Part B, Mathematics & Statistics Part B, and across the MPLS Division.

Having said this, as in Table 1 the proportion of first class degrees in Mathematics alone awarded (39.84%) was high, and the proportion of II.2 and below degrees in Mathematics awarded (15.48%) was low, compared to the guidelines.

Setting and checking of papers and marks processing

Requests to course lecturers to act as assessors, and to act as checker of the questions of fellow lecturers, were sent out early in Michaelmas Term, with instructions and guidance on the setting and checking process, including a web link to the Examination Conventions. The questions were initially set by the course lecturer, in almost all cases with the lecturer of another course involved as checker before the first drafts of the questions were presented to the examiners. Most assessors acted properly, but a few failed to meet the stipulated deadlines (mainly for Michaelmas Term courses) and/or to follow carefully the instructions provided.

The internal examiners met at the beginning of Hilary Term to consider those draft papers on Michaelmas Term courses which had been submitted in time; consideration of the remaining papers had to be deferred. Where necessary, corrections and any proposed changes were agreed with the setters. The revised draft papers were then sent to the external examiners. Feedback from external examiners was given to examiners and to the relevant assessor for response. The internal examiners at their meeting in mid Hilary Term considered the external examiners' comments and the assessor responses, making further changes as necessary before finalising the questions. The process was repeated for the Hilary Term courses, but necessarily with a much tighter schedule.

Due to the Pandemic, Exam Papers were revised and set to be open book. Camera ready copy of each paper was signed off by the assessor, and then submitted to the Examination Schools.

Candidates accessed and downloaded their exam papers via the Weblearn system at the designated exam time. Exam responses were uploaded to Weblearn and made available to the Exam Board Administrator 48 hours after the exam paper had finished.

The process for Marking, marks processing and checking was adjusted accordingly to fit in with the online exam responses. Assessors had a short time period to return the marks on the mark sheets provided. A check-sum was also carried out to ensure that marks entered into the database were correctly read and transposed from the mark sheets.

All scripts and completed mark sheets were returned, if not by the agreed due dates, then at least in time for the script-checking process.

A team of graduate checkers, under the supervision of Barbara Galinska and Elle Styler, sorted all the marked scripts for each paper of this examination, cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way a number of errors were corrected,and each change was signed by one of the examiners who were present throughout the process. A check-sum was also carried out to ensure that marks entered into the database were correctly read and transposed from the marks sheets.

Throughout the examination process, candidates were treated anonymously, identified only by a randomly-assigned candidate number, until after all decisions on USMs, degree classes, mitigating circumstances notices to examiners, prizes, and so on, had been finalized.

Standard and style of papers

It was noted at the Final Exam Board meeting that the papers 3.4 Algebraic Number Theory and 4.3 Distribution Theory were set too easy this year. These papers will need to be reviewed, especially if the exams are held as open-book again.

Timetable

Examinations began on Monday 31st May and finished on Tuesday 22nd June.

Determination of University Standardised Marks

We followed the Department's established practice in determining the University standardised marks (USMs) reported to candidates. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part A performances of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage). Working with the data for this population, numbers N_1, N_2 and N_3 are first computed for each paper: N_1, N_2 and N_3 are, respectively, the number of candidates taking the paper who achieved in Part A average USMs in the ranges $[69.5, 100]$, $[59.5, 69.5)$ and $[0, 59.5)$.

The algorithm converts raw marks to USMs for each paper separately. For each paper, the algorithm sets up a map $R \to U$ (R = raw, U = USM) which is piecewise linear. The graph of the map consists of four line segments: by default these join the points $(100, 100)$, $P_1 = (C_1, 72)$, $P_2 = (C_2, 57)$, $P_3 = (C_3, 37)$, and $(0, 0)$. The values of C_1 and C_2 are set by the requirement that the number of I and II.1 candidates in Part A, as given by N_1 and N_2, is the same as the I and II.1 number of USMs achieved on the paper. The value of C_3 is set by the requirement that P_2P_3 continued would intersect the U axis at $U_0 = 10$. Here the default choice of *corners* is given by U-values of 72, 57 and 37 to avoid distorting nonlinearity at the class borderlines.

The results of the algorithm with the default settings of the parameters provide the starting point for the determination of USMs, and the Examiners may then adjust them to take account of consultations with assessors (see above) and their own judgement. The examiners have scope to make changes, either globally by changing certain parameters, or on individual papers usually by adjusting the position of the corner points P_1, P_2, P_3 by hand, so as to alter the map raw $\to$ USM, to remedy any perceived unfairness introduced by the algorithm. They also have the option to introduce additional corners. For a well-set paper taken by a large number of candidates, the algorithm yields a piecewise linear map which is fairly close to linear, usually with somewhat steeper first and last segments. If the paper is too easy or too difficult, or is taken by only a few candidates, then the algorithm can yield anomalous results—very steep first or last sections, for instance, so that a small difference in raw mark can lead to a relatively large difference in USMs. For papers with small numbers of candidates, moderation may be carried out by hand rather than by applying the algorithm.

Following customary practice, a preliminary, non-plenary, meeting of examiners was held two days ahead of the plenary examiners' meeting to assess the results produced by the algorithm alongside the reports from assessors. The examiners reviewed each paper and report, considered whether open book examination process affected candidates and reviewed last year's statistics. The examiners discussed the preliminary scaling maps and the preliminary class percentage figures. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low.

Table 2 on page 8 gives the final positions of the corners of the piecewise linear maps used to determine USMs.

In accordance with the agreement between the Mathematics Department and the Computer Science Department, the final USM maps were passed to the examiners in Mathematics & Computer Science. USM marks for Mathematics papers of candidates in Mathematics & Philosophy were calculated using the same final maps and passed to the examiners for that School.

Comments on use of Part A marks to set scaling boundaries

None.

Mitigating Circumstance Notice to Examiners

A subset of the examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of the individual notices to examiners. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate.

The full board of examiners considered 40 notices in the final meeting. The Board also received a total number of 16 MCEs carried over from the 2020 Part A Final Mathematics Exam Board. The examiners considered each application alongside the candidate's marks and the recommendations proposed by the Part A 2020 Exam board. The outcomes for these have been recorded on a spreadsheet report on Mitigating Circumstances Notice to Examiners from Part A.

All candidates with certain conditions (such as dyslexia, dyspraxia, etc.) were given special consideration in the conditions and/or time allowed for their papers, as agreed by the Proctors. Each such paper was clearly labelled to assist the assessors and examiners in awarding fair marks.

Table 2: Position of corners of the piecewise linear maps

Paper	P_1	P_2	P_3	Additional Corners	N_1	N_2	N_3
B1.1	16.72;37	29.1;57	42.70	50;100	6	19	10
B1.2	9.54;37	16.6;57	43.6;57	50;100	13	22	10
B2.1	10;37	19;57	38;72	50;100	14	15	4
B2.2	14;58	27.8;73	50;101		12	17	4
B3.1	10.05;37	20;57	39;72	50;100	19	26	6
B3.2	13.04;39	22.7;59	39.2;74	50;102	7	11	3
B3.3	13.04;39	22.7;59	39.2;74	50;102	13	7	3
B3.4	12.52;37	21.8;57	42.8;72	50;100	19	20	5
B3.5	10;37	27;57	40;72	50;100	11	16	8
B4.1	7.75;37	13.5;57	36;72	50;100	13	19	5
B4.2	7;37	17;57	33;72	50;100	9	15	4
B4.3	10.86;37	18.9;57	41;70	50;100	5	3	1
B4.4	17.64;37	28;60	32.2;72	50;100	9	10	7
B5.1	11.43;37	19.9;57	30.4;72	50;100	9	10	7
B5.2	9.02;37	15.7;57	32;72	50;100	21	24	8
B5.3	7;37	18;57	26;72	50;100	14	11	5
B5.4	9.19;37	19;57	31;72	50;100	14	10	5
B5.5	10;36	21;56	40.2;71	50;99	19	27	13
B5.6	13.37	27.9;57	38.4;72	50;100	10	11	4
B6.1	13;37	27;57	40;72	50;100	5	10	5
B6.2	16;37	34.2;57	41;72	50;100	2	4	1
B6.3	8.1;37	17;57	40;70	50;100	1	8	0
B7.1	10;37	24;57	37.6;72	50;100	14	12	5
B7.2	13.56;37	22;57	35;6;72	50;100	10	7	6
B7.3	7.47;37	15;57	28;72	50;100	9	8	5
B8.1	9.88;37	18;57	40;72	50;100	16	36	9
B8.2	10;37	16;57	37;72	50;100	14	45	15
B8.3	9.88;37	17.2;57	41;72	50;100	14	45	15
B8.4	16.89;36	29;56	44;69	50;99	6	25	7
B8.5	12.35;37	21.5;57	43;72	50;100	9	25	4
BSP	2000;100				2	7	3
SB1	14.19;37	24.7;57	49.5;72	66;100	10	35	6
SB1	34;100				10	35	6
SB2.1	12.52;37	20;57	42.8;72	50;100	15	38	7
SB2.2	13.37	24.1;57	43.6;72	50;100	16	36	11
SB3.1	9;37	16.1;57	35.6;72	50;100	24	48	13
SB3.2	18.38;37	29;57	41;70	50;100	3	4	5
SB4	22;54	50;100			0	0	0

B. Equality and Diversity issues and breakdown of the results by gender

Table 3: Breakdown of results by gender

Class	Number								
	2021			2020			2019		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	13	38	51	18	55	73	13	46	59
II.1	22	36	58	28	38	66	18	49	67
II.2	4	14	18	3	10	13	5	15	20
III	1	0	1	1	3	4	1	3	4
P	0	1	1	0	0	0	0	2	2
F	0	0	0	0	1	1	0	0	0
Total	50	107	15	50	107	157	37	114	151
Class	Percentage								
	2021			2020			2019		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	32.5	43.18	75.68	36	51.4	43.7	35.14	40.35	39.07
II.1	55	40.91	95.91	56	35.51	45.76	48.65	42.98	44.37
II.2	10	15.91	25.91	6	9.35	7.68	13.51	13.16	13.25
III	2.5	0	2.5	2	2.8	2.4	2.7	2.63	2.65
P	0	0.93	0.93	0	0	0	0	0	0
F	0	0	0	0	0.88	0.66	0	0	0
Total	100	100	100	100	100	100	100	100	100

Table 3 shows the performances of candidates broken down by gender.

Table 4: Rank and percentage of candidates with this or greater overall USMs

Av USM	Rank	Candidates with this USM and above	%
87	1	1	0.78
83	2	2	1.56
82	3	6	4.69
81	7	7	5.47
80	8	9	7.03
78	10	12	9.38
77	13	16	12.5
76	17	20	15.62
75	21	23	17.97
74	24	27	21.09
73	28	33	25.78
72	34	41	32.03
71	42	44	34.38
70	45	50	39.06
69	51	53	41.41
68	54	61	47.66
67	62	69	53.91
66	70	77	60.16
65	78	86	67.19
64	87	89	69.53
63	90	95	74.22
62	96	97	75.78
61	98	101	78.91
60	102	106	82.81
59	107	109	85.16
59	107	109	85.16
58	110	112	87.5
57	113	117	91.41
56	118	118	92.19
55	119	121	94.53
53	122	122	95.31
52	123	123	96.09
51	124	125	97.66
50	126	127	99.22
48	128	128	100

C. Detailed numbers on candidates' performance in each part of the examination

Data for papers with fewer than six candidates are not included.

The number of candidates taking each paper is shown in Table 5.

Individual question statistics for Mathematics candidates are shown below for those papers offered by no fewer than six candidates.

Paper B1.1: Logic

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.19	21.19	3.63	31	0
Q2	14.58	14.58	6.49	26	0
Q3	14.89	14.89	7.29	9	0

Paper B1.2: Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.59	9.59	5.49	14	3
Q2	19.43	19.43	5.18	35	0
Q3	14.67	14.67	5.48	45	1

Paper B2.1: Introduction to Representation Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.61	15.61	3.62	31	0
Q2	20.79	20.79	5.61	14	0
Q3	9.50	9.50	4.55	10	0

Paper B2.2: Commutative Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.28	9.28	4.25	25	0
Q2	13.55	13.55	7.16	20	0
Q3	12.16	12.16	5.99	19	0

Table 5: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
B1.1	33	35.45	9.81	65.3	16.59
B1.2	47	31.64	10.56	68.11	11.55
B2.1	33	31.88	8.48	68.15	9.38
B2.2	33	22.24	9.87	64.79	15.58
B3.1	51	33.94	8.27	70.24	11.02
B3.2	22	38	8.04	74	13.04
B3.3	24	35.25	8.77	74.04	10.06
B3.4	44	35.8	8.82	69.3	9.71
B3.5	36	35	8.82	69.56	13.02
B4.1	33	27.76	8.77	67.55	8.27
B4.2	28	26.5	8.76	66.32	10.35
B4.3	9	35.33	11.18	68.44	13.08
B4.4	5	-	-	-	-
B5.1	20	25.25	6.32	63.85	10.79
B5.2	49	26.86	8.13	67.43	10.3
B5.3	29	24.76	8.76	67.69	13.24
B5.4	28	26.96	7.18	66.96	10.08
B5.5	41	33.27	9.27	66.96	10.08
B5.6	27	32.96	8.82	65.59	14.11
B6.1	20	34.7	9.64	66.1	16.37
B6.2	7	36.57	8.32	65	13.3
B6.3	6	34.5	9.22	68.5	7.97
B7.1	29	34.07	7.42	69.62	11.33
B7.2	21	31.05	8.11	67.38	11.92
B7.3	21	21.86	8.7	63.1	14.3
B8.1	35	31.6	8.89	66.83	9.66
B8.2	19	32.26	9.17	71.16	10.69
B8.3	42	28.9	8.76	63.88	8.87
B8.4	25	36.32	7.96	63.24	10.42
B8.5	33	35.64	7.96	69.76	10.66
SB1	8	33.88	11.21	70.25	6.11
SB2.1	23	33.83	7.24	66.39	6.44
SB2.2	22	34.59	5.68	65.64	5.98
SB3.1	46	27.33	7.81	65.72	8.69
SB3.2	2	-	-	-	-
SB4	1	-	-	-	
CS3a	1	-	-	-	-
CS4b	2	-	-	-	-
BO1.1	8	-	-	68.88	9.66
BO1.1X	8	-	-	68.13	10.59
BEE	5	-	-	76.8	7.89
BSP	8	TBC	TBC	TBC	TBC
102	13	-	-	65.08	4.5
127	9	-	-	61.33	23.67
129	1	-	-	-	

Paper B3.1: Galois Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.92	14.92	5.71	49	0
Q2	-	-	-	-	2
Q3	14.83	14.83	3.82	46	0

Paper B3.2: Geometry of Surfaces

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.43	20.43	3.69	14	0
Q2	20.07	20.07	4.94	14	0
Q3	16.81	16.81	5.52	16	0

Paper B3.3: Algebraic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.13	16.13	5.03	16	0
Q2	18.00	18.00	4.80	24	0
Q3	19.50	19.50	3.55	8	1

Paper B3.4: Algebraic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.29	14.29	5.21	14	0
Q2	19.56	19.56	3.66	39	0
Q3	17.49	17.49	5.34	35	0

Paper B3.5: Topology and Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.45	18.45	5.03	20	0
Q2	16.44	16.44	5.27	36	0
Q3	17.39	18.69	5.38	16	2

Paper B4.1: Functional Analysis I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.57	14.57	4.49	28	0
Q2	13.14	13.14	6.07	29	0
Q3	14.11	14.11	3.76	9	0

Paper B4.2: Functional Analysis II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.39	14.39	4.76	28	0
Q2	11.38	11.38	5.86	16	0
Q3	12.15	13.08	5.79	12	1

Paper B4.3: Distribution Theory and Fourier Analysis: An Introduction

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.78	17.78	6.16	9	0
Q2	17.56	17.56	5.32	9	0

Paper B4.4: Fourier Analysis and PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	-	-	-	4	0
Q2	-	-	-	3	0
Q3	-	-	-	3	0

Paper B5.1: Stochastic Modelling and Biological Processes

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.10	12.10	3.61	20	0
Q2	-	-	-	3	0
Q3	11.61	11.88	3.27	17	1

Paper B5.2: Applied PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.61	13.61	4.60	38	0
Q2	11.76	11.76	4.86	29	0
Q3	14.77	14.77	5.04	31	0

Paper B5.3: Viscous Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11	11	5.72	7	0
Q2	12.82	13.15	5.14	27	1
Q3	11.92	11.92	5.19	24	0

Paper B5.4: Waves and Compressible Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.56	13.56	6.49	16	0
Q2	14.71	14.71	2.61	24	0
Q3	11.56	11.56	3.69	16	0

Paper B5.5: Further Mathematical Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.92	15.92	6.26	13	0
Q2	16.38	16.38	3.70	32	0
Q3	17.11	17.11	6.13	37	0

Paper B5.6: Nonlinear Systems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.77	19.48	6.11	52	1
Q2	-	-	-	5	0
Q3	15.08	15.08	3.93	24	2

Paper B6.1: Numerical Solution of Differential Equations I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.10	16.10	6.97	10	0
Q2	18.33	18.33	2.48	18	1
Q3	17.17	18.45	6.49	11	0

Paper B6.2: Numerical Solution of Differential Equations II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	-	-	-	6	0
Q2	18.71	18.71	5.31	7	0
Q3	-	-		1	0

Paper B6.3: Integer Programming

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.50	16.50	6.83	6	0
Q2	-	-	-	4	0
Q3	-	-	-	2	0

Paper B7.1: Classical Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.75	14.75	3.59	24	
Q2	18.30	18.30	4.61	27	0
Q3	20.00	20.00	5.33	7	0

Paper B7.2: Electromagnetism

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.90	15.90	4.82	20	0
Q2	15.79	15.79	4.34	14	0
Q3	-	-	-	6	0

Paper B7.3: Further Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.73	10.73	3.43	15	0
Q2	9.11	9.25	3.76	8	1
Q3	12.61	13.18	5.46	17	1

Paper B8.1: Martingales through Measure Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.96	15.96	4.82	27	0
Q2	17.79	17.79	4.69	28	0
Q3	11.80	11.80	4.99	15	0

Paper B8.2: Continuous Martingales and Stochastic Calculus

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.75	17.75	4.36	16	0
Q2	-	-	-	5	0
Q3	15.06	15.06	5.87	17	0

Paper B8.3: Mathematical Models of Financial Derivatives

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.55	15.55	4.81	33	0
Q2	12.80	13.17	5.64	24	1
Q3	14.26	14.26	4.91	27	0

Paper B8.4: Communication Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.33	16.14	5.64	14	1
Q2	18.21	18.21	4.29	19	0
Q3	19.76	19.76	3.80	17	0

Paper B8.5: Graph Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.55	18.55	5.47	33	0
Q2	16.84	16.84	3.72	32	0
Q3	-	-		1	0

Paper SB2.1: Foundations of Statistical Inference

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.82	18.82	2.50	22	0
Q2	15.90	15.90	6.56	10	0
Q3	13.93	14.64	5.48	14	1

Paper SB2.2: Statistical Machine Learning

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.90	15.90	4.22	21	0
Q2	18.56	18.56	2.99	16	0
Q3	18.57	18.57	3.26	7	0

Paper SB3.1: Applied Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.11	13.56	5.92	43	2
Q2	13.92	13.92	3.13	38	0
Q3	13.18	13.18	5.21	11	0

Assessors' comments on sections and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. The examiners have not included assessors' statements suggesting where possible borderlines might lie; they did take note of this guidance when determining the USM maps. Some statistical data which can be found in Section C above has also been removed.

B1.1: Logic

Question 1 This question was attempted by nearly all students. Part (a) was generally done correctly with no issues. In (b), parts (i) and (ii) were generally done with no problems, but in (iii) many students missed that derivability required an additional step (simplest via completeness). Part (c) was generally done without problems. The derivation in (d) was the most challenging part of the question, but was completed effectively by most students via a variety of different paths, with the result that this question overall saw in general strong scores.

Question 2 This question was attempted by a substantial majority. Part (a) was generally well done with students demonstrating the adequacy in the first case and recognizing the reason for inadequacy in the second. The reverse direction in (b) caused some difficulty with a number of students not seeing how to complete the argument. The logical validity in (c) was generally done thoroughly. The (non-)elementary equivalence in (d) part (i) was generally recognized though many students did not give a formula in the given language. Part (ii) required a carefully set up compactness argument. Quite a few gave a careful and correct argument, though a number gave erroneous purported demonstrations that the Theorem is elementary.

Question 3 This question was attempted by a considerably smaller number of students. Part (a) was generally done well. Part (b) was often done a bit carelessly, perhaps under time pressure, with sentences in (i) and (ii) not saying quite the right thing, or with no regard to the requirement (II). Part (iii) was again very mixed with some students giving a clearly argued (negative) answer, while other answers were cursory. Part (i) of (c) was an exercise left in lectures and was generally well done. It also represented a small hint for part (ii) and most students did remark the appropriate check when using axioms of type A4. This part also was often done well, sometimes by somewhat circuitous routes.

B1.2: Set Theory

This was the second year in which exams were done remotely, in open-book format, and the first year in which all teaching was affected, or devastated, by the Covid-19 pandemic. People were working, without, for much of the time, any possibility of personal, face-to-face contact, under circumstances which varied from suboptimal to appalling. Under these circumstances, it is this assessor's opinion that all students who battled through the year and succeeded in putting pen to paper for the exam deserve to be congratulated. It has only added to the difficulties of this year that under the remote open-book format, arrangements are unavoidably less secure.

It was decided that exam questions should include less bookwork and be a little harder. The result, for this paper, was that the range of marks was very wide. Some candidates, who got very low marks, might have done rather better in a year in which they could demonstrate some competence with the subject by doing bookwork (reproducing proofs of theorems etc).

The individual questions:

Question 1. This was judged by the candidates to be a difficult question; fewer people did it and there were fewer very high marks.

In part (a), there were many good answers. The commonest error was in proving that any maximal element of a suitable family of partial orders was a total order. People took a non-total order and tried to prove that it was not maximal by extnding it. Quite often the relation that they extended it to, was not transitive. In many cases, people would have discovered the error, and perhaps corrected it, if they'd attempted to prove that the order was indeed a partial order.

In (b)(iii), many people got confused about which sets where elements of which other sets, a crucial question when comparing the "altered" version of the transfinite recursion theorem with the original.

The difficulties that people found with parts (c) and (d) were unsurprising. Perhaps the approach to take with these was to think for a few minutes very hard about what the proposed theorems are actually saying and what they mean, before deciding that the proof of the statement in (c) is little different from what it would have been if the order had been a well-ordering, while the statement in (d) is absurd.

Question 2. This is about ordinals. The commonest serious error in part (a) was to confuse ordinal and cardinal arithmetic, and assert that ω^ω is uncountable. In addition, some people unthinkingly assumed that both distributive laws held, which gave rise to errors in calculating $(\omega+3)(\omega+1)$.

In (b)(i), it's worth making a fuss of the fact that $\sup A$ is countable, because the statement that a countable union of countable sets is countable does depend on the Axiom of Choice.

The errors and difficulties that people found in (b)(ii) and (iii) were unsurprising. The way around this was probably for a candidate to think about fixed-point theorems they might know in topology (it's reasonable to think of "tidy", or more conventionally, normal functions as continuous). The Tarski Fixed Point Theorem is unhelpful here, partly because it doesn't apply, and partly because the proof in the lecture notes follows a different pattern, not depending on any notion of continuity.

Question 3. This is about cardinals. Part (a) follows a pattern often seen on this paper over the years. The most difficult parts were (iv) and (v), where in both cases the answer is $2^{\aleph_0}$; the Continuum Hypothesis turns out to be irreleveant to this, though many candidates incorrectly either believed it necessary or assumed it without comment and perhaps unconsciously.

The main point in (b)(i) is that if λ is a limit cardinal, the existence of κ_λ depends on the axiom scheme of Replacement; in set theories without replacement, κ_ω need not exist.

A common route to success in part (b)(ii) was to prove by induction on α that $\alpha \leq \kappa_\alpha$, and then use Hartogs' Theorem.

0.1 B2.1 Introduction to Representation Theory

Question 1. Very popular with the students, with all but three having a good go at it. Part (a) was done very well by the majority, although only a few people gave convincing and correct arguments for the injectivity of the map in part (a)(iv): the equation ae = 0 does not immediately imply $a = 0$ without using the $AeA = A$ condition. Part (b)(i) was done very well by most people, but unfortunately (ii) proved to be too hard. Having clearly understood the connection between the character table of a finite group and the structure of its group algebra would have been helpful here: the quaternion group has four distinct linear characters defined over the reals, which have to contribute a direct factor of R^4 in the group ring RQ_8 — as an R-algebra. A moment's thought then shows that this direct factor must be equal to the eRQ_8 in the question. So there are a total of sixteen idempotents in eRQ_8, one for each subset of the standard basis for R^4.

Question 2. Attempted by roughly two thirds of the students. It was mostly done well. Those students who managed to correctly compute the conjugacy classes in part (a) are the ones who tended to get a very high mark for this question overall. Unfortunately, the converse was also true: conjugacy class structure is very important in the representation theory of finite groups and any minor errors committed when calculating conjugacy classes tend to have substantial effects on what follows. For part (c) there were some quick proofs of the fact that G/N is S_3 such as observing that otherwise it would be C_6 hence abelian and hence the derived group would be contained in N which contradicted the conclusion of part (b).

Question 3. Least popular and only a few students managed to get to the end of it. Arguably, part (c) was the most difficult question on this paper. One common mistake was to confuse the two different notions of "character", namely the trace of a representation and a group homomorphism with values in the group of non-zero complex numbers. In part (a)(iv), several people thought the subgroup H that one had to identify was equal to N. In fact, H is the stabiliser of the character of N under the G-action on the set of characters of N that was considered in part (a)(i): whilst this stabiliser always contains N, it is in general strictly larger than N.

B2.2: Commutative Algebra

All questions were about equally popular.

Question 1: In (b), several candidates mistakenly concluded that P(t) is a minimal polynomial in the sense of field theory. This is not correct, as K—R is not a field extension. In (c), very few students gave a full proof of injectivity. The proof follows from a careful analysis of the non-divisibility of a homogenous polynomial by $y^2 - x^2 - x^3$.

Question 2: in (e) (i) very few candidates noticed that the existence of maximal elements can be deduced without appealing to Zorn's lemma. The existence follows from the noetherian property directly.

Question 3: In (f), very few candidates actually verified that the proposed upper bound for the partial order actually has the right properties.

B3.1: Galois Theory

Q1 and Q3 were by far the most popular questions.

Question 1.In (e) of Q1, very few students exploited the fact that all the quadratic extensions of a given finite field coincide. Most solutions involved quadratic residues. The other parts of the question were generally solved satisfactorily.

Question 2 was solved satisfactorily by the (very few) students who considered it, with the exception of (a) (i). For this part of the question, notice that it follows from the definitions that the minimal polynomial of alpha over M_1 is both a divisor and a multiple of $P(x)$, and hence must coincide with it.

Question 3. Here quite a few students struggled with (b). Note that the minimal polynomial of rho over Q(omega) must be of degree 1 or 5 by Kummer theory. It cannot be 1 because the degree of Q(omega) over Q is not divisible by 5. Hence it must be five. The other parts of the question were generally solved satisfactorily, although part (d) was a hurdle for some.

B3.2: Geometry of Surfaces

Question 1. The majority of candidates did this question well and got a high mark (typically 22). Often (d) was not well done, but this only cost candidates 1–2 marks. Lower scoring candidates usually did the atlases in (a) badly, for example, writing down an 'atlas' for T^2 consisting of two discs, which only covered $T^2 \setminus \{\text{point}\}$. The usual way of proving (b) was to make a chart W on $X\#Y$ by gluing $U \setminus \{\text{disc}\}$ and $V \setminus \{\text{disc}\}$ for charts U, V on X, Y, and candidates often failed to explain why W was diffeomorphic to an open subset of $\mathbb{R}^2$ (e.g. by 'inverting' $V \setminus \{\text{disc}\}$ around the boundary circle of the disc in $\mathbb{R}^2 \cong \mathbb{C}$ by $z \mapsto \bar{z}^{-1}$ and taking $W = U \setminus \{\text{disc}\} \cup \bar{z}^{-1}[V \setminus \{\text{disc}\}]$).

Question 2. Most candidates got (a) correct or nearly correct. For (b),(c), usually either candidates understood the question well and got a high total mark (20–24), or understood it badly and got a low total mark (6–11). Candidates often lost a mark in (b) by forgetting to list the branch points. Few candidates answered the final part of (c) really well.

Question 3. Many candidates made calculation mistakes in (a),(b), which hampered their answers to (c). In (a), having got the equation for principal curvatures in the form $(E\lambda - L)(G\lambda - N) = 0$ as $F = M = 0$, a depressing number of candidates multiplied out and used the quadratic formula rather than just writing down $\lambda = L/E, N/G$. In (b) a common mistake was to fail to reparametrize the curve by arc-length, leading to the wrong answer.

For the last part of (c), I was hoping for the answer that the displayed equation comes from a Gauss–Bonnet Theorem with boundary, with a missing term $2\pi\chi(R)$, which is zero since $\chi(R) = 0$ as R is an annulus. Disappointingly, nobody said this. Some candidates cut the annulus into a rectangle and used Gauss–Bonnet for a disc with polygon boundary, which got full marks for this part. For (d), I was hoping for the answers: (i) $\kappa_g = 0$ by (b), or fixed curve of a reflection symmetry; (ii) by reflection symmetry; and (iii) this is a straight line, and geodesic (length-minimizing) in $\mathbb{R}^3$, so is also geodesic in X. Few candidates saw all three.

The spread of marks, from low to high, was greater than I would have expected in a non-COVID year. I think this is because of the minimization of bookwork for open-book exams, candidates that did not understand the material well tended to go wrong, and score poorly thereafter.

B3.3 Algebraic Curves

Question 1: This question had 17 attempts, of which 9 were in the 18-25 mark range and 4 in the 13-17 range. Most candidates got the existence of the 1-parameter pencil of conics including three singular conics, though very few spotted the noncalculational proof for the singular examples (they are just the three line pairs through the four points). The last part of (a) proved difficult, despite the hint. Quite a few candidates got confused between complex symmetric and Hermitian matrices, and hardly any saw the significance of there being 3 singular conics in the pencil, giving 3 distinct eigenvalues for diagonalisation. There was also confusion about how conics transformed under linear maps–many thought it was conjugation rather than A 7! PAPt. Part (b) was much better done, and most candidates were confident with using Bezout's theorem. There were some neat arguments here, using the symmetries of the curve to avoid calculations.

Question 2: This question was very popular and every candidate attempted it, with 15 achieving marks in the 18-25 range and 8 getting marks in the 13-17 range. Characteristic p examples had come up in problem sheets, but several candidates did get into difficulties with the example here. There were some excellent answers however. In part (b) the Hessian calculations were generally done well, and most candidates found the $x + y + z$ factor, showing reducibility of the Hessian curve. Some did not spot that the Hessian in fact completely factors into 3 linear factors.

Question 3: This was the most sophisticated question, dealing with Riemann-Roch, and only received 9 attempts. However most of these were very good, with 7 in the 18-25 range and the other two in the 13-17 range. It was good to see that most candidates were quite expert with using Riemann-Roch, and interpreting the results.

B3.4: Algebraic Number Theory

Question 1: was answered by 17 (out of 52) candidates; Parts (a),(b),(c)(i),(iv) were answered well, but some had trouble with Parts (c)(ii),(iii),(v), and in particular showing that (2) is the product of three distinct prime ideals.

Question 2: was answered by 46 (out of 52) candidates, and it was done to a very high standard, with only a few difficulties, mainly in (c)(iii), where a common mistake was to talk about the principal ideal of → before establishing that → is in the ring of integers. Question 3 was answered by 41 (out of 52) candidates; part (a) was generally well answered; the most common error was to forget to check that [P2] was not [P3]2; part (b) was very well answered; in part (c), a common error was to assume that three elements of order 2 must generate $C_2 \times C_2 \times C_2$, without checking whether they are independent; many candidates did not attempt part (d), but those who did answer it made good progress with it.

B3.5 Topology and Groups

Question 1: (31 attempts) This question tested the understanding of the base change for the fundamental group. Points we gained were the homotopies where clearly described. Part c proved more difficult though some excellent solutions were given. Generally the question was well done with several perfect scores.

Question 2: (53 attempts) Generally students made good attempts. The most difficult parts were part b (ii) and (iv). Instead of considering maps out of the group to detect the properties in question some candidates tried to use Tietze transformations which invariably led to mistakes. In part c points were lost because candidates thought retractions had to be homotopy equivalences.

Question 3: (30 attempts) Candidates who had absorbed the last section of the course and had geometric intuition did generally well. In the first question, only few summarised the bookwork on the Galois correspondence well.

B4.1: Functional Analysis I

Question 1 was solved by most of the students. The first part of 1a) as well as the proof that the space is complete for $\alpha \geq 0$ was generally well solved.

The proof of incompleteness of Y_α for negative α caused more problems than expected as many students did not realise that e.g. a cut-off sequence of a constant function could be used as non-convergent Cauchy sequence. The last part of a) was designed to be challenging and while many students obtained some partial points only few students gave a complete answer. Many students solved 1b) very well and obtained full marks. The first part of 1c) was generally well solved by students attempting this part of the question, though few realised that the easiest way of arguing would be to view E as kernel of a bounded linear operator. The last part of c) was more challenging but several students came up with examples of finite sequences with sum 0 which converge in ℓ^p to an element for which the required sum does not converge.

Question 2 was solved by about two thirds of the students. The first part of question 1) turned out to be more difficult than anticipated. While most students argued by contradiction, many did not realise that they needed to adjust the corresponding bookwork proof by normalising the usual sequence (x_n) with $\|Tx_n\| > n$ and $\|x_n\| = 1$ by setting e.g. $\tilde{x}_n = \frac{1}{\sqrt{n}}x_n$ instead of $\tilde{x}_n = \frac{1}{n}x_n$. Nearly all students realised that the second part of 2a) was a standard consequence of the definition of a bounded linear operator. The second part of 2 was generally well solved. In the first part some points were lost as students did not explain why the resulting function was continuous or as students tried to prove that the operator norm for the specific example was 3 rather than 3/2. About half of the students realised that since all functions in the image are differentiable away from 0 the map cannot be surjective and many also realised that there is a connection between the map being injective and the set of zeros of g. The first and second part of 2c) could have been solved by small adjustments of proofs of two corollaries of Hahn-Banach, but not many students realised this and instead tried to either prove these parts from scratch or by somehow trying to apply such corollaries rather than their proofs. Conversely most of those who attempted the last bit of 3c) realised that they needed to use Hahn-Banach though few discussed why an extension with norm 8 exists if and only if the original map has operator norm no more than 8.

Question 3 was only solved by about a quarter of the students. The first part was a standard exercise on dual operators and spectrum and was well solved. The second part was an application of bookwork and while most students realised that one inequality simply follows by triangle inequality, not many used that as $1 + \|T\| \in \sigma(I + T)$ one can obtain the other inequality from the fact that the spectrum is contained in the corresponding closed ball. Part c) was designed to be the most challenging part of the question and that proved to be true. While several students realised that $CD^n - D^nC = nD^{n-1}(Id + D)$, and several also remarked that $(Id + D)$ is invertible, no one managed complete the contradiction argument by arguing that the norm of the left hand side of $(CD^n - D^nC)(Id + D)^{-1} = nD^{n-1}$ is bounded from above by a fixed multiple of $\left\|D^{n-1}\right\|$. The first part of Question 3c) follows as the kernel of functionals has codimension 1 and hence a finite intersection of such kernels must be non-empty if the space is infinite dimensional, though not many students solved this question. Most students who attempted the last part of 3c) realised that this is an application of a corollary of Hahn-Banach.

B4.2: Functional Analysis II

Question 1: This question was tried by all candidates. In (a), most candidates had no problem except for showing that $(X, \langle\cdot,\cdot\rangle_T)$ is a Hilbert space implies (*) holds, where one typically used either the Open mapping theorem or the Inverse mapping theorem. In (b), a number of candidates had difficulties in realising that the space to apply the Riesz representation theorem to is $(X, \langle\cdot,\cdot\rangle_T)$, rather than the original space. Those who saw this might fail to check that the linear map they considered was indeed bounded with the new inner product. Part (c) was attempted by a slightly smaller number of candidates with variable degrees of success. Successful candidates saw right away that $P^2 = P$ and $P^* = P$, but a quicker solution perhaps involves checking directly that P is the identity on ImT (which is closed in view of (*)) and is trivial on $(ImT)^\perp$.

Question 2: This question was tried by about half of the candidates. Those who recognised the relevance of (a)(i) with the proof of the Open mapping theorem handled this part without an issue. Half of those who tried (a)(i) went on to handle (a)(ii) successfully. (b)(i) and the first half of (b)(ii) were handled well mostly. Many candidates saw the idea of the second half of (b)(ii) but did not realise the subtlety that the linear functionals on L^2 are represented by a function in L^2 rather than a function in X. To use the first half, one would need to use the completeness of the trigonometric system. (b)(iii) was attempted by a very small number of candidates who all had some right ideas.

Question 3: All parts of this question were attempted by about half of the candidates. (a) was handled well with some minor exceptions. For (b), the majority of the candidates tried to contradict the closedness of ImA if A

was infinite-dimensional. Only a fraction realised that this part could be handled rapidly with the help of the Baire category theorem. For the last part of (c), successful candidates usually considered the restriction of A to a suitable finite dimensional subspace. The first part of (d) was handled mostly reasonably. Only one candidate successfully handled the last part of (d), where one considered the extension of a finite dimensional matrix in Jordan form.

B4.3: Distribution Theory and Fourier Analysis: An Introduction

There were only 11 candidates taking the exam and their performances were generally quite good. No issues were reported and the marking scheme was used throughout.

Question 1: Most did well on this question, though nobody got the full 25 marks. Marks were lost by all candidates in (a)(iii) when proving that the concrete distributional derivative has order one (and not merely at most one). Some marks were also lost in (b), where in some cases it appears that the candidates ran out of time. However, most of the candidates that attempted this part had the correct approach.

Question 2: Most did well on this question, though nobody got the full 25 marks. Part (a) tests the candidates understanding of support of a distribution. This did not cause any difficulties in the majority of cases and there were some excellent solutions from a few of the candidates. Marks were generally lost in part (b), where some candidates failed to find particular solutions or presented incomplete arguments.

Question 3: Nobody attempted this question.

B4.4: Fourier Analysis and PDE's

There were only 7 candidates taking the exam and, though it was a challenging paper, their performances were quite good. No issues were reported and the marking scheme was used throughout.

Question 1: The majority attempted this question and got full or close to full marks on part (a). Marks were lost by many candidates in the second half of (b) that asked to find the distributional limit of the imaginary part of the holomorphic function. Part (c) went surprisingly well and those who attempted it got almost full marks on it.

Question 2: About half the candidates attempted this question. It went reasonably well and there was one really excellent solution obtaining the full 25 marks. Part (a) is an elaborate variant of a calculation done in lectures for the Bessel kernel and those who had paid attention to that would have had no difficulties completing it. Part (b) tests the Plancherel theorem in a particular example and was done well by those who attempted it. Part (c) requires candidates to localize their result from (b) and deduce from (a) that the differential operator in question is hypoelliptic.

Question 3: About half the candidates attempted this question. Judging from the outcome it was the hardest question as all who attempted it struggled with the last part. Parts (a)(i), (ii) are largely variants of bookwork and went well. Candidates who lost marks in part (a) did so on (a)(iii), mainly for presenting a sloppy argument. Part (b)(i) did not present a problem for those who attempted it, but its implementation in (b)(ii) turned out to be a challenge and nobody got it quite right.

B5.1: Stochastic Modelling and Biological Processes

The exam structure was comparable to the B5.1 exam in the previous year (2020) with three questions covering similar parts of the course as in 2020. Comparing to the year 2020, the average raw mark across the whole cohort went down, which has to be understood in the context of the unusual way the examining was conducted in 2020, when students were required to drop some of their Part B exams, because of pandemic arrangements. Some self-selection resulted so that the candidates opting to take the 2020 exam were students with a very good understanding of the course material, while in 2021, the submitted results were combinations of some very sound solutions together with some less successful solution attempts.

Some self-selection also applied to the popularity of each question in this year exam (2021). Most of the candidates opted to submit Questions 1 and

3. There were only four solutions handed in for Question 2, which was not necessarily “more difficult”, but it did cover the material from the last third of the course. In fact, while the most popular Questions 1 and 3 have almost the same average raw mark, the candidates who handed in Question 2 did so well, that the average raw mark of Question 2 was higher than the raw marks of Questions 1 and 3. In Question 2, candidates adapted the derivations from the Lecture Notes to slightly modified problems. For the introductory questions, they could apply the formulas for the diffusion coefficient and the velocity auto-correlation function from the Lecture Notes, while for more difficult questions, they demonstrated their good understanding of the course material by modifying the derivations from the Lecture Notes.

In Questions 1 and 3, most of the submitted solutions achieved at least 10 raw marks in each of these questions, which illustrates that even the weaker candidates did study the corresponding part of the material and were able to apply the concepts from the Lecture Notes. For more advanced topics, candidates knew which part of the Lecture Notes contained the relevant part of the theory. The difference between better and worse solutions of the more advanced part was that some candidates were able to notice that the theory cannot be applied as it was in the Lecture Notes and that some modifications were necessary.

B5.2: Applied PDEs

Question 1: (a) Most students got the characteristic ODE + initial values right and also the solution; a few failed to notice that the problem was stated in conservation form and used f where f′ would have been appropriate. Many students also used the Jacobian / envelope method as starting point for discussing the question when the classical solution ceases to exist, but some then failed to realise what to look for (ie to take an appropriate min/max), but a small number got to the final result. So this was one of the harder parts of the equation. (b) was generally well done, though some students did use other expression in the causality inequality for the characteristic speeds than values of f′. Some then got the algebra right to find the range and argument for the final statement of the question correctly. (c) Quite students got a, b, c and alpha, beta right, but some forgot to test invariance of the initial condition. Some students got the ODE wrong in (ii), others didn’t finish the question, but a reasonable number managed to get ‘the other’ solution for w. Some forgot to discuss the impact of the initial conditions. (iii) was only attempted by a few students and obtaining the shock position was a challenge.

Question 2: Candidates found this question challenging. Part (a)(i) was generally done well, and many students got the relations for alpha and beta right, but some had trouble due to algebraic errors. The resulting ODE was required the application of integrating factors which was found by some students. This was critical to answer other aspects of (a) and also (b). Part (b) the relation for (d omega/dt) was done well, but expressing this in self-similar variables was more challenging. Only very few candidates tackled (b)(ii).

Question 3: This question was generally well done. Typical instances were mistakes and omissions were made are as follows: In (a), the meaning of the delta function was interpreted physically though no physical context was implied. Also, quite a number of candidates got beta wrong (frequently beta=0 instead of -1/2) which had knock on effects regarding the normalisation condition. In (b), the connection with (a) was not made properly and the alpha dependence of the Green's function made without clear derivation, frequently leading to erroneous results. In (c), the verification of the normalisation condition was omitted. In (d), which was correctly done by many candidates, sometimes the notation was unclear regarding the arguments of the Green's function or some of the contributions dropped (e.g. sum of only three free-space Green's functions).

B5.3: Viscous Flow

There were a few good solutions, some nearly perfect, but the overall standard was low. Candidates seemed to have insufficient familiarity with routine calculations and thus ran out of time before reaching the more challenging parts of questions. Several candidates started questions with dimensionally incorrect scalings, e.g. scaling a velocity with a length. Question 1 was much the least popular, but the average marks for the three questions were quite similar.

Question 1 – only a quarter of candidates attempted this question.

Several attempts for part (a) used a material volume instead of the required space-fixed volume. The energy flux across the boundary of a space-fixed volume includes an advective term of **u** times the energy density.

In part (b) almost everyone wrote the viscous term incorrectly as $\mu\partial_{yy}u$ instead of $\partial_y(\mu\partial_y u)$. The latter comes from the divergence of the stress σ with $\sigma_{xy} = \mu\partial_y u$. However, full marks were given for correct solutions in later parts that followed from this incorrect starting point.

In part (c) the velocity scale should be $U = GH^2/\mu_0$. The parameter ϵ is half the Prandtl number times the ratio of the kinetic energy $\frac{1}{2}\rho U^2$ to the thermal energy $\rho c_v T_w$ based on the velocity scale $U = GH^2/\mu_0$ and the wall temperature T_w. The ratio of kinetic to thermal energy characterises the relative change in temperature through viscous heating.

In part (d) there is no steady solution if both boundaries are insulating. The heat generated viscously cannot leave the channel containing the fluid.

In part (e) the correct answer is $\hat{u}_1 = (1/72)(1 - \hat{y})^2(\hat{y}^4 + \hat{y}^2 + 8\hat{y} - 14)$. The corresponding solution for the incorrect problem with μ outside the derivatives is longer.

Question 2 – almost all candidates attempted this question.

In part (a) candidates were expected to scale $U = \sqrt{FL}$ to derive dimensionless equations involving only the Reynolds number. Some candidates left U arbitrary, while others balanced the body force with the viscous force, not looking ahead to (b).

In part (b) some candidates omitted viscosity completely in what are supposed to be viscous boundary layer equations. The y-component of the body force does not appear in the leading-order y-momentum equation, which is $(1/\delta)\partial_y p = O(1)$.

In part (c) it is easiest to deduce that the pressure gradient vanishes using the far field behaviour, where the flow is at rest, then rewrite the equations from (b) using a streamfunction. Some candidates formed the vorticity equation first, which takes longer. Several candidates omitted the body force, and even the pressure, right from the start, then re-inserted a body force at the end to match the given result.

Many candidates kept $\nabla^2 T$ with both T_{xx} and T_{YY} instead of applying the boundary layer scaling. A few kept T_{xx} only, which caused later difficulties in finding a similarity solution. Some candidates replaced $\mathrm{D}T/\mathrm{D}t$ with $\partial_x T$.

It is easiest to write $T = T_\infty + (T_w - T_\infty)\hat{T}$ so that $\hat{T} = 1$ on $\hat{Y} = 0$ and $\hat{T} \to 0$ as $\hat{Y} \to \infty$, then choose the velocity scale U to make the dimensionless body force just $\hat{T}$.

In part (d) one needs both the x-momentum equation and the temperature equation $\hat{\psi}_{\hat{Y}}\hat{T}_{\hat{x}} - \hat{\psi}_{\hat{x}}\hat{T}_{\hat{Y}} = \mathrm{Pr}^{-1}\,\hat{T}_{\hat{Y}\hat{Y}}$ to determine $\alpha = 3/4$, $\gamma = 1/4$ and $\beta = 0$. The last is determined by the temperature boundary condition $\hat{T} = 1$ on $\hat{Y} = 0$. The other equation satisfied by f and g is

$$-\frac{3}{4}ff'' + \frac{1}{2}f'^2 = f''' + g.$$

Part (e) received only one complete solution. When $\mathrm{Pr} \ll 1$ the given expression for g_c solves the equation $g'' + (3/4)\,\mathrm{Pr}\,fg' = 0$ in the boundary layer at infinity with $\eta = O(1/\mathrm{Pr})$ and $f(\eta) \approx f(\infty)$. The same expression reduces to $g_c(\eta) = 1 + O(\mathrm{Pr})$ when $\eta = O(1)$. This solves the leading-order problem $g'' = 0$ with $g(0) = 1$ and matches to the solution in the boundary layer at infinity.

Question 3 – this was the next most popular

Part (a) was mostly done well, though a few candidates started with dimensionally incorrect scalings.

In part (b) candidates were encouraged to use incompressibility to solve for $\hat{u}$ in terms of $\hat{w}$, which was specified to be independent of $\hat{r}$. Instead, almost all candidates first solved for $\hat{u}$ in terms of $\hat{p}$, as in lectures, then solved for $\hat{w}$. Full marks were given if this was done correctly. However, many candidates tried to use the "constant" of integration for $\hat{w}$ to impose the boundary condition $\hat{w} = \pm\hat{h}_{\hat{t}}$ on $\hat{z} = \pm\hat{h}$. No choice of constant can satisfy both boundary conditions. The constant must vanish, keeping $\hat{w}$ an odd function of $\hat{z}$. The boundary conditions then determine the elliptic equation that $\hat{p}$ must satisfy. No integration across the layer is needed to find the velocity field.

To establish *global* mass conservation candidates were expected to show that the rate of change in the dimensionless volume $2\pi\hat{h}$ of fluid between the two discs equals the volume flux across the edge at $\hat{r} = 1$. This is the only part that requires an integration across the layer. It was not enough to observe that $\nabla \cdot \mathbf{u} = 0$. A few candidates tried to use a different geometry from lectures with a constant height and a varying radius.

Part (c) attracted few attempts. Some candidates restarted the problem from scratch instead of balancing the imposed force with the integral of the pressure from part (b) across the discs. The velocity scale is $U = \delta^2 F/(\mu R)$, and the height scale is $H = h(0)$. For the last part one needs to solve $\mathrm{d}\hat{r}/\mathrm{d}\hat{t} = \hat{u}(\hat{r}(\hat{t}), 0, \hat{t})$. The fluid element initially at $\hat{r} = \hat{r}_0$, $\hat{z} = 0$ reaches the edge $\hat{r} = 1$ at time $(3\pi/16)(\hat{r}_0^{-8/3} - 1)$ that goes to infinity as $\hat{r}_0 \to 0$.

B5.4: Waves and Compressible Flow

Question 1: The fairly routine exercise in deriving the forced wave equation in part (a) was generally well handled, although many candidates did not adapt the methodology covered in lectures and therefore lost marks for missing out parts of the argument (such as the justification for the existence of a velocity potential for the perturbed flow). The application of standard methodology to the new wavemaker problem in part (b) was generally well done, though many candidates lost marks for inaccurate algebraic manipulations of the roots of an auxiliary equation. There were many good attempts at part (c)(i), but only a few spotted the correct secular solution in part (c)(ii).

Question 2: The standard Stokes waves problem in part (a)(i) caused few difficulties apart from an occasional lack of accuracy. The algebraic manipulations stumped the majority in the rider in part (a)(ii) despite them mirroring a similar example in the lecture notes. The exercise in separation of variables in part (b)(i) was generally well done, though only a handful made the most judicious choice for the form of general solution of the ODE for $F(x)$ and hence avoided substantial unnecessary algebraic manipulations. Only a few candidates sought the correct secular solution in part (b)(ii) and many repeated their analysis of part (b)(i) to derive invalid solutions.

Question 3: Many more attempts than expected ran into algebraic difficulties in part (a), which was supposed to be a relatively routine manipulation of the Rankine-Hugoniot conditions supplemented by one additional constraint. The non-standard punctured-membrane problem in part (b)(i) was not well done, with many attempts failing to justify the domains of the different solutions or to give sufficient explanation. Very few candidates made any headway with part (b)(ii) and there were numerous inaccurate or incomplete sketches of the characteristic diagram. Overall this question was found much harder than anticipated.

B5.5: Further Mathematical Biology

Question 1. This question was attempted by relatively few candidates. Most answered (a) correctly, and derived the correct equations in (b)(i). However, many did not fully justify the assumptions underlying the model derivation. Part (b)(ii) was less well-answered, with only a small number of candidates fully explaining why the population tends to a steady state under the given parameter constraint. Part (c)(i) was mostly well-answered. Almost all candidates found (c)(ii) difficult, and many did not manage to establish the given result.

Question 2. This question was attempted by the vast majority of candidates. Parts (a) (b)(i) were generally well-answered. In (b)(ii) there was a mistake in Equation (2), which should have read $\mathrm{d}U/\mathrm{d}\eta = -UV$. Most candidates spotted this error and so derived the correct equations (often with the signs reversed, $V \mapsto -V$). In the remainder of (b) most candidates chose to work with the equations as stated in the exam paper (so that Equation (2) read $\mathrm{d}U/\mathrm{d}\eta = +UV$), and equivalent marks were awarded.

Question 3. This question was generally relatively well-answered. Marks were often lost in (b) for not clearly showing that the stated model is non-dimensional. Many candidates could not correctly a further condition required for patterning.

B5.6: Nonlinear Systems

Q1. This was a popular question, attempted by most candidates. On the whole it was answered well, and there were some very good answers.

Q2. This was an unpopular question, with only a handful of attempts, most of which were poor.

Q3. This was the second most popular question, attempted by all but a few candidates. There were one or two good answers, and many partial answers. Some candidates struggled with finding the centre manifold of the discrete system, although most seemed to know what they should be doing in principle. The most common mistake was forgetting to make sure the centre manifold was tangent to the centre subspace.

B6.1: Numerical Solution of Differential Equations I

Question 1
This question on the numerical approximation of Hamiltonian systems was attempted by roughly half of the candidates who took the exam. There was one complete answer and four strong attempts. There were also four-five weak attempts by candidates who have, evidently, not revised this part of the syllabus despite the fact that there was a similar problem on one of the problem sheets, which was also discussed in the intercollegiate classes.

Question 2
This was very popular and was attempted by most of the candidates. Part (a) was uniformly well done. In part (b) several candidates confused second-order consistency with second-order convergence, and having checked second-order consistency jumped to the conclusion that the method is second-order convergent, without invoking Dahlquist's theorem and verifying zero-stability of the method. Most of the candidates managed to complete part (c) of the question, either by calculating the roots of the quadratic stability polynomial and showing that they belong to the open unit disc in the complex plane for all values of $h\lambda \in (-\infty, 0)$ or by using Sturm's criterion. Part (d) was completed (or almost completed) by four candidates only; a frequent oversight was the false assertion that

the fact that (y_n) is a monotonically decreasing sequence of positive real numbers converging to 0 was the immediate consequence of the absolute stability of the multistep method; whereas the absolute stability of the method only implies that $|y_n| \to 0$ as $n \to \infty$ with no guarantee of monotonicity of the sequence (y_n).

Question 3

This question on the numerical approximation of the one-dimensional heat equation by the explicit Euler method was attempted by roughly half of the candidates. The question was generally well done, although some of the candidates overlooked the fact that the function $u \in (-\infty, \infty) \mapsto \arctan u \in (-\pi/2, \pi/2)$ is Lipschitz-continuous, with Lipschitz constant equal to 1, which is necessary for completing the proof of the desired inequality in part (c).

B6.2: Numerical Solution of Differential Equations II

This seemed an acceptable open-book paper. Only one candidate attempted question 3 on finite difference methods for hyperbolic conservation laws; they scored a high mark.

All candidates attempted question 2 on the rotated five-point finite difference formula for the Dirichlet problem for the Poissson equation and there were several good and two poor marks. Several candidates were somewhat muddled in trying to put together a convergence proof using the maximum principle.

All but one candidate attempted question 1 on finite differences for a two-point boundary value problem including a first order term and the resultant consideration of the linear algebra. No candidate produced a completely acceptable answer to the second (and main) part of (b): most obtained the finite difference inequality, but none sought to solve the corresponding linear constant coefficient 2nd order finite difference equation to obtain a desired example of violation of the maximum principle for the given parameter value.

There were 2 poor scripts; these candidates did not seem to have really come to terms with the material of the course. Their arguments were often confused with inadequate definition of terms and/or description of what they were trying to achieve.

B6.3: Integer Programming

Question 1 was most popular with 12 attempts, highest mark of 25 and a mean of 15.8. There was a considerable spread of marks with a standard deviation 7.8. Some candidates struggled with the IP formulation of a 1-tree, or with the definition of a 1- tree, despite this being book work. Depending on the formulation chosen in part c) there were multiple different approaches possible for part d), drawing on different parts of the course. Most candidates tried to use total unimodularity. Question 2 was the second most popular with 9 attempts with marks between 14 and 23. This question seems to have been slightly easier than the other two with a mean of 18.8 and a standard deviation of 3.2, as the derivation of Gomoroy cuts and the dual simplex steps were a routine task. Part a) required attention to detail, as the problem (IP) was formulated in inequality constrained form rather than in terms of equality constraints. Likewise, in part d) an argument was required as to why the identified extended cover inequalities dominate all other extended cover inequalities. No one gained full marks, though some candidates came very close. Question 3 saw 7 attempts resulting a marks in the range from 9 to 19, with a mean of 15.1 and a standard deviation of 4.3. The greedy algorithm in part b) required an adaptation of the standard greedy algorithm discussed in the course to be able to deal with integral rather than binary constraints on the decision variables. In part d) most candidates gave an extended formulation of a graph with $O(b * u_1 * \ldots * u_n)$ nodes, while there exists a much more parsimonious solution with only O(nb) nodes.

B7.1: Classical Mechanics

1. This question was popular but proved quite challenging for most candidates. In part b few were able to correctly identify the coordinate transformation between the Lagrange multiplier formulation and the reduced Lagrangian despite this being in the notes. The calculation of the effective potential in c(i) was often incorrect with the conserved quantity being used to eliminate variables in the Lgrangian as opposed to the Hamiltonian. In c(ii) few were able to identify the normal reaction with the Lagrange multiplier terms discussed in part b.

2. Question 2 was the most popular and well done by many, although many were unable to perform the calculations accurately in part b.

Few identified the trajectory as that between the unstable equilibria $\omega = (0, \pm A, 0)$.

3. Question 3 was relatively straightforward for those few that had got to that late stage of the course although it was nevertheless possible to get bogged down in calculation.

B7.2: Electromagnetism

There where 24 attempts this year. Most students seem to understand the main ideas and basic content of the lecture course. Many students indeed did very well, though many marks were lost due to computational errors.

Question 1 was on electrostatics. There were a number of errors in part (b)(ii). Some were computational but others did not set up the boundary conditions correctly or forgot to add the potential for the charge the center of the shell. Many students did not attempt the last part of part (c).

Question 2 This problem involved a charged rotating infinite cylinder. There where many computational errors in the computation of the magnetic field (part (c)).

Question 3 This question was about the computation of the time varying electromagnetic field due to a dipole with time dependent charges joined by a thin conducting wire. Though this question was not as popular it was not more difficult than Q1 or Q2 because enough information was *given*. The last part seemed particularly hard.

B7.3 Further Quantum Theory

Question 1 Most candidates attempted this question on one-dimensional scattering, with the

first part being well answered in general. In the second part, the value of the re ection probability could be easily worked out using the continuity of the probability current, but most candidates instead proceeded to direct computation. There were also many mistakes regarding the form of the wave function in the classically forbidden region, leading to erroneous results. Very few candidates made a significant attempt on the third part, though it could be solved even without finishing the previous part by taking advantage of the simplified boundary condition at x = 0 when the potential W ! 1. The fourth part was touched on schematically by a few candidates, but very few worked it out in detail. The Bohr-Sommerfeld estimate of the number of bound states could be done independently of the rest of the question, but for the most part this was unattempted.

Question 2 This question about perturbation theory and addition of angular momentum was the least popular. The

first part required an overview of the structure of Hydrogen atom stationary states with respect to both orbital/spin angular momenta and total angular momentum. This was mostly answered well, though there were already a number of small mistakes having to do with the range of values of various quantum numbers. The second part introduced perturbations, but in the first subpart the perturbations were exactly diagonalisable in one of the two standard bases for the Hydrogen atom stationary states, so no actual perturbation theory was required.This was recognised by a good number of candidates, though the accuracy of the calculations was mixed. In particular, the freedom to use rotations to set the magnetic field **B** to lie in the x3 direction and the important identity $\mathbf{S} \cdot \mathbf{L} = \mathbf{12}(\mathbf{J2} - \mathbf{L2} - \mathbf{S2})$ were missed in many cases. The second and third subparts required implementing

first order degenerate perturbation theory in two different ways, but in both cases the degeneracy turned out to be irrelevant (all o-diagonal terms of the perturbation vanished in the natural bases). Few candidates got far in these subparts.

Question 3 This WKB problem was another popular one. The first part required observing that the boundary conditions at the two classical turning points are different, with one leading to a phase shift due to the connection formulae and the other setting the wave function to zero. Many candidates missed this and gave the Bohr-Sommerfeld condition for the case of two smooth turning points. In the second part, the main problem was to

x the various relative coefficients of the WKB solutions in the allowed and forbidden regions, including an important factor of two that comes from the connection formula. There were many errors in this part, with only a few candidates giving the completely correct wave functions. The third part followed from a simple application of the virial theorm to the case of a linear potential, but without using the virial theorem there would be a very challenging integral to perform and this was a sticking point in many cases. Finally, the last part could be solved independently and gave a wave function that was an Ai type Airy function. Candidates had to do a change of variables to get things into the form of an Airy equation, then identify the Ai rather than the Bi function as the appropriate one to be normalisable, and extract energy levels from the boundary condition at x = 0. Many candidates did the first of these, fewer the second, and still fewer the third.

B8.1: Probability, Measure and Martingales

Questions 1 and 2 were equally popular, while question 3 saw 50% less attempts in comparison. The standard of answers was also comparable between the questions, with question 2 having the highest average, question 1 slightly below and question 3 slightly lower still.

Students often lost some marks on the first easy parts of the problems due to lack of care. For example, in Q1.a.i) students were asked to "carefully state any properties of the conditional expectation that you use" but many failed to do so and lost one or two marks. Similarly, when checking that a process is a martingale some forgot to check the easy conditions of being adapted and integrable, or failed to give any reasons for simple computations they made. Q2.a) was testing the understanding of the proof of Jensen's inequality and it often proved challenging. However, one direction was obvious here and some forgot to state it losing a mark. Q3.a.i) confused some who were trying to use countable additivity for an uncountable union of sets.

The last parts of questions 1 and 3 often caused difficulties. Some were confused about conditions for L^1 convergence vs L^2 convergence. There were few fully correct answers arguing that M is uniformly integrable in Q3.b.iv).

B8.2: Continuous Martingales and Stochastic Calculus

Question 1 was generally well done, and was attempted by around 80% of candidates. Some candidates had trouble with part aii, as they either simply proved that the process X is bounded in L^2, or asserted that uniform integrability of X was sufficient to prove convergence. For part c, the key is to use Lévy's characterization, which means that one simply needs to verify that V is a local martingale with quadratic variation $\langle V \rangle_t = t$. Attempting to prove the result directly, using the increments of V, is difficult.

Question 2 was attempted by a quarter of candidates. Parts a and b were generally well done, with some candidates making the error of implicitly assuming that the martingales start at zero. Part c posed more difficulty;

in part cii many candidates did not successfully show that the integral $\mathbb{H} \blacklozenge \mathbb{W}$ converges, with many only showing that the quadratic variation is bounded. The key is to look at the 'tail' of the sum, that is, to show that $\sum_{j>k} H^{(j)} \bullet W^{(j)} \to 0$ in $\mathcal{H}^2$.

Question 3 was attempted by around 80% of candidates. The initial parts of the question were well done, with some candidates forgetting to specify the distribution of X fully in part aii (either omitting to state the mean or variance), or not clearly justifying why the integral is Gaussian. In part b, some candidates attempted to use the solution of part ai to write down the quadratic variation of X, which is misleading (as t appears inside the stochastic integral). The convergence in part bii was generally well done, provided candidates noticed that Y is a nonnegative local martingale. Part c posed some difficulty, with candidates not clearly explaining why the non-convergence of X implies that Y must also not converge (it depends on s having a continuous monotone inverse, not simply on s being continuous). Part cii depends on applying the optional stopping theorem to Y (rather than X, which is not a (local) martingale).

B8.3: Mathematical Models of Financial Derivatives

Question 1 was taken by most candidates. The question essentially covered the basic material for the course, but required it to be re-derived for the Bachelier model, rather than the BlackScholes model. Some students struggled as they simply reproduced the calculations for the BlackScholes model verbatim, rather than making the necessary adjustments for the different underlying dynamics.

Question 2 was taken by around two thirds of candidates. Many students gave a full and detailed summary of the BlackScholes argument in part (a), rather than stating the definition of a replicating portfolio, and outlining its role (we use delta-hedging to construct a replicating portfolio, and hence obtain prices using the law of one price). For part (b), many candidates did not recall the definition of a forward price, and so were not able to give a clear statement of the construction of a risk-free contract using stocks and forwards, or assumed that the forward contract was the same as the contract they were seeking to price (with certain payoff 1). For the latter parts of the question, many answers did not use the function V derived in part (c) to full effect, instead re-derived the value (for different choices of p) in each of the later sections. This led to incomplete answers (presumably due to time pressure).

Question 3 was taken by around half of the candidates. Part (a) of the question was generally well don, as it was similar to examples shown in lectures. Some candidates had difficulty with (aiii), demonstrating a lack of familiarity with working with payoff diagrams. Part (b) was generally less well done. Common issues were incorrectly assuming the option in (bi) was a simple barrier option, not giving a clear answer for part (biii), or simply not submitting answers to these questions (presumably due to time pressure).

Overall the standard of answers was very varied, with some candidates giving answers to most sections, while others struggled significantly with later parts of the exam.

B8.4: Information Theory

Question 1 Few students made mistakes in part (a), which are on basic inequalities. Some students failed in the proof of the convexity in part (b.i), and part (b.iii) is the one to distinguish candidates' understanding on convexity and the definition of divergence. Surprisingly, most candidates did well in part (c) on the optimisation.

Question 2 Part (a) should be an easy disproof of the optimality by simple counter examples, while two candidates answered the question with a big theory. In part (b), many candidates lost one or two points in different subparts because they didn't mention that new codes they constructed are prefix code. For this part, (b.iii), (b.v) and (b.vi) are most difficult. There are answers different from the standard one, some of which are better and simpler, but some are lengthy while not very relevant. Part (c) and (d) are based on the conclusions in part (b) and answers can be easily reshaped from lecture notes.

Question 3 This questions sounds the easiest among the 3 questions in the paper, the average marks is 2 points higher than the other two. Part (a) was on the definition of DMC and capacity, and part (b) was on the application of a simple inequality. Almost all candidates got full marks in these two parts. Part (c) are on three cases of the given DMC, which sounds not as challenging as it was expected, and most candidates provided perfect answers. Part (c) was also not difficulty, but many students had not explained different measures of the error rate.

Summary: Marks in this paper are high, many students get 40+ out of 50. It shows that students have a good understanding on the subject. But it may also shows that the paper was not challenging enough. It clearly differentiates poor candidates from good ones, but it is not very successful to differentiates strong candidates from good ones.

B8.5: Graph Theory

No comments

BO1.1: History of Mathematics

Both the extended coursework essays and the exam scripts were blind double-marked. The marks for essays and exam were reconciled separately. The two carry equal weight when determining a candidate's final mark. The first half of the exam paper (Section A) consists of six extracts from historical mathematical texts, from which candidates must choose two on which to comment; the second half (Section B) gives candidates a choice of three essay topics, from which they must choose one. The Section B essay accounts for 50% of the overall exam mark; the answers to each of the Section A questions count for 25%.

Throughout the course, candidates were invited to analyse historical mathematical materials from the points of view of their 'context', 'content', and 'significance', and these were the three aspects that candidates were asked to consider when looking at the extracts provided in Section A of the exam paper. Indeed, most candidates chose to use these as subheadings within their answer. This is entirely acceptable, although some candidates had a tendency to place details under the wrong headings.

The Section A questions 1–6 were attempted by 3, 5, 6, 4, 0, and 2 candidates, respectively. The only specific extracts that candidates had certainly seen before were those of questions 2, 5 and 6 (though they had seen the diagram in question 4, if not the accompanying text). Questions 1 and 2 related to core material from the lecture course, namely calculus and pre-calculus. They were generally well done, though some candidates omitted to mention an important feature of the extract in question 2: that it presents an algebraic, rather than geometric, approach to the early calculus. Although question 3 was the most popular question in this section, there were some misinterpretations of Euler's slightly convoluted prose: he had circumstantial evidence that his conjecture was true (although we now know that it is not). A minor point not mentioned here by some candidates was that the solutions sought were specifically solutions in *radicals*. Question 4 was generally well done, though in places lacked some of the earlier background to the development of the understanding of complex numbers. Questions 5 and 6 were the harder questions of this section: question 5 because it related only to the material of a single lecture (hence its being rejected by all candidates), and question 6 because there is a limit to what can be said under 'significance', so it was necessary to get this just right in order to score high marks.

This year, the Section B questions were generally better done than those of Section A, though some answers could have been organised better. Questions 7–9 were attempted by 3, 2, and 5 candidates, respectively. Question 7 was probably the easiest of this section, being a standard form of question, with plenty of material from the lecture course to draw upon. Question 8, by contrast, was probably the hardest, since this is a topic (the influence of Euclid aside) that received only a little attention during the course. The most popular question here, question 9, was generally well done, with several candidates first considering what counts as pure or applied mathematics, although some of the categorisations were questionable. After addressing this point, not all candidates brought the discussion back adequately to the mathematicians themselves.

The standard of the extended essays was on the whole very high, with good use of source materials despite the current difficult circumstances. There were some excellent examples of engaging literary style, which the assessors read with genuine enjoyment. Ideas were expressed well, and showed good historical understanding and innovative thinking. Most candidates displayed their appreciation of the need to locate the studied texts within their proper context, rather than simply seeking to interpret them in light of modern ideas. This was combined with a good technical understanding. A common pitfall within the essays was a tendency to be a little unclear in places about the meaning of the word 'analysis', a crucial point in an essay about the changing meaning of the word. The term sometimes seemed to be used implicitly in its modern sense, with a risk of distorting the historical story. Some candidates failed to mention infinite series in their essays, although these are arguably a key part of the story via the important link between having functions as a basis for analysis, and the use of infinite series to establish certain quantities *as* functions.

Statistics Options

Reports of the following courses may be found in the Mathematics & Statistics Examiners' Report.

SB1.1/1.2: Applied and Computational Statistics

SB2.1: Foundations of Statistical Inference

SB2.2: Statistical Machine Learning

SB3.1: Applied Probability

SB3.2: Statistical Lifetime Models

SB4: Actuarial Science

Computer Science Options

Reports on the following courses may be found in the Mathematics & Computer Science Examiners' Reports.

CS3a: Lambda Calculus & Types

CS4b: Computational Complexity

Philosophy Options

The report on the following courses may be found in the Philosophy Examiners' Report.

102: Knowledge and Reality

127: Philosophical Logic

129: Early Modern Philosophy

E. Comments on performance of identifiable individuals

1.Aggregation of marks for the award of the classification on the successful completion of Parts A and B

Classification for a candidate was determined through the following method:

- 10 units at Part A (counting A2 as a double-unit and, for candidates offering 6 long options, two of the long option papers as half-units)
- 6 units (or equivalent) at Part B.

The two average USMs will be:

1. The relative weightings of the Parts is as follows:
   (a) The weighting of Part A is 40%.
   (b) The weighting of Part B is 60%.
2. The relative weightings of the Parts is as follows:
   (a) The weighting of Part A is 100%.
   (b) The weighting of Part B is 0%.

The first class Strong Paper Rule says that to get a first class degree the candidate must have:

(a) average USM $\geq$ 69.5;

(b) at least 6 units in Parts A and B with USMs $\geq$ 70;

(c) at least 2 units in Part B with USMs $\geq$ 70.

The analogous rules apply for II.1 and II.2 degrees. The examiners considered all candidates near each borderline who had been caught by the Strong Paper Rule, that is, who satisfied (a) but failed (b) or (c), and so were due to receive the lower degree class. For one such candidate at the I/II.1 borderline the examiners decided to suspend the examination conventions, and placed the candidate in the first class.

F. Names of members of the Board of Examiners

- **Examiners:**

  Prof Nick Trefethen (Chair)
  Prof Paul Dellar
  Dr. Neil Laws
  Dr. Kevin McGerty
  Prof. Jim (James) Oliver Prof. Cornelia Drutu
  Prof Marco Schlichting (External)
  Prof Anne Skeldon (External)

- **Assessors:**
  Prof. Andreas Muench
  Prof. Andrew Dancer
  Prof. Andy Wathen
  Prof. Chris Beem
  Dr Chris Hollings
  Prof. Damian Rössler
  Prof. Dominic Joyce
  Prof. Endre Suli
  Dr Hanqing Jin
  Prof. Jan Kristensen
  Prof. Jan Obloj
  Prof. Jim Oliver
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  Prof. Konstantin Ardakov
  Prof. Lionel Mason
  Prof. Luc Nguyen
  Prof. Melanie Rupflin
  Dr Paul Balister
  Prof. Paul Dellar
  Prof. Radek Erban
  Prof. Raphael Hauser
  Dr Robin Knight
  Prof. Ruth Baker
  Prof. Sam Cohen
  Prof. Ulrike Tillmann

Prof. Victor Flynn
Prof. Xenia de la Ossa